

Magnetoelectrics in Disordered Topological Insulator Josephson Junctions

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We theoretically study the coupling of electric charge and spin polarization in an equilibrium and nonequilibrium electric transport across a two dimensional Josephson configuration comprised of disordered surface channels of a three dimensional topological insulator. In the equilibrium state of the system we predict the Edelstein effect, which is much more pronounced than its counterpart in conventional spin orbit coupled materials. Employing a quasiclassical Keldysh technique, we demonstrate that the ground state of system can be experimentally shifted into arbitrary macroscopic superconducting phase differences other than the standard ‘0’ or ‘ π ’, constituting a ϕ_0 -junction, solely by modulating a quasiparticle flow injection into the junction. We propose a feasible experiment where the quasiparticles are injected into the topological insulator surface by means of a normal electrode and voltage gradient so that oppositely oriented stationary spin densities can be developed along the interfaces and allow for directly making use of the spin-momentum locking nature of Dirac fermions in the surface channels. The ϕ_0 -state is proportional to the voltage difference applied between the injector electrode and superconducting terminals that calibrates the injection rate of particles and, therefore, the ϕ_0 shift.

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I. INTRODUCTION

Topological state of matter has been a striking topic during the past decade and has attracted extensive attention both theoretically and experimentally^{1,2}. The study of nontrivial electronic structure topology goes back to the prediction of the time-reversal symmetry protected electronic states bound to the contact of two semiconductors with mutually inverted bands³ and exotic particles in liquid He⁴, which now offer promising prospects to practical platforms such as spintronics and quantum computation^{1,2}. The extensive and growing research efforts in this context have led to the exploration of topological insulators (TIs)^{5–9}, for instance. Surface channels of a three dimensional topological insulator are conductors³ while bulk material itself is insulator with a gap in its band structure. In contrast to numerous theoretical works so far^{1,2}, these surface channels may not be fully ballistic and can host unavoidable defects or other quasiparticle scattering resources such as impurities^{10–12}. The recent experiments have also revealed this fact that the conduction of the surface states are limited with a finite resistance^{13,14}. This key observation clearly proves the fundamental importance of considering the contributions and influences of disordered motions of moving particles in the surface channels^{10–14}.

The surface channels may be employed as an appealing opportunity that can support spin-momentum locked modes, namely, the orientation of particle’s spin is locked to its momentum direction^{1,2}. The motion of low energy quasiparticles in these surface states are governed by a Dirac equation, meaning the particles’ velocity is energy independent in low energy bands. The interplay

of the Dirac particles with superconductivity can result in rich and intriguing physics such as particle to antiparticle conversion^{15,16}. While the surface states themselves may not be superconducting, they may however adopt superconducting properties by proximity to a superconducting electrode^{17–22}. Therefore, such a platform can provide the unique possibility to materialize the interplay of relativity and superconductivity in laboratory and possibly make use of its advantages in functional devices and technological ways²³.

Motivated by the recent experimental progresses in fabricating TI-superconductor (S) heterostructures^{17–22}, we here study one of the manifestations of interplay between the superconductivity and spin-momentum locking nature of Dirac fermions on the surface of a 3D TI: The coupling between spin and charge degrees of freedom in a two dimensional Josephson junction made of disordered surface states of a three dimensional TI under equilibrium and nonequilibrium conditions.

In the equilibrium state, we predict that the system responds to a dc supercurrent flowing across the junction by developing a stationary spin density oriented along the junction interfaces. This phenomenon can be viewed as a direct magnetoelectric effect i.e. the Edelstein effect. In the *conventional* intrinsic spin orbit coupled metals, analogous effect was first theoretically predicted in Refs. 24 and 25 and later observed experimentally in Refs. 26 and 27. It was shown that spin polarization can be produced in the spin orbit coupled metals by externally applied electric field.^{24–27} The magnetoelectric polarizability was also discussed in the normal phase of topological insulators^{1,2,28}. We note that similar magnetoelectric effect was also predicted for bulk superconductors and su-

perconducting heterostructures by means of supercurrent due to the spin orbit interactions^{29–31}. Here we develop a theory to this phenomenon in S-TI-S heterostructures and show that a topological insulator can support much more pronounced signatures than those of conventional spin orbit coupled metals.

We also demonstrate an inverse coupling between the charge and spin degrees of freedom in S-TI-S heterostructures. In particular, it is shown that the injection of quasiparticle current at the middle of junction in opposite directions yields spontaneous magnetizations with opposite orientations in each segment. The current-induced magnetization is directly coupled to the phase difference between the superconducting terminals. This phenomenon renders the minimum of junction free energy into a superconducting phase difference between the S terminals $\phi = \phi_0$, which is generally unequal to 0 or π . The ϕ_0 is proportional to the voltage difference between the injector electrode and superconducting terminals and inverse of Fermi velocity on the surface states. ϕ_0 -junctions were also predicted in ferromagnetic Josephson junctions with spin orbit interaction^{32–36} and may be viewed as an inverse magnetoelectronic effect in such hybrids³⁷.

Nearly all of the past theoretical works on the spin orbit coupled Josephson structures involves ferromagnetism or an external magnetic field as key ingredients to establish the magnetoelectric effect. In this paper, however, we demonstrate that an internal Zeeman-like term can be generated and electrically controlled in S-TI-S heterostructures (in the absence of any ferromagnetic elements or an external magnetic field) simply by means of quasiparticles injection. The Zeeman-like term causes an anomalous phase shift ϕ_0 in the supercurrent that can be electrically controlled via the quasiparticle flow injection generated by a voltage difference.

So far, the electric control of the superconducting critical temperature^{38–40}, the magnitude of the Josephson current^{41,42}, switching between 0 and π states^{43–48}, and spontaneously accumulated spin currents⁴⁹ have been discussed in hybrid structures with spin orbit coupling. The electric control of the anomalous phase shift was experimentally realized in superconducting heterostructures on the basis of a quantum wire²³. This kind of controll also was theoretically proposed in S/silicene/S heterostructures³⁶. Nonetheless, in these proposals, the anomalous phase shift occurs only in the presence of an externally applied magnetic field or ferromagnetic elements. Our findings offer the ability of integrating the superconducting topological insulator nanostructures into electronic circuits without the requirement to apply magnetic field or making use of magnetic elements.

The paper is organized as follows. In Sec. II, we summarize the theoretical framework used and basic assumptions made for studying a disordered topological insulator Josephson junction under nonequilibrium conditions. The direct magnetoelectric effect in S-TI-S heterostructures is presented in Sec. III. The electrically controllable

inverse magnetoelectric effect is discussed in Sec. IV. We finally summarize concluding remarks in Sec. V.

II. NONEQUILIBRIUM KELDYSH TECHNIQUE

Numerous systems in laboratory deal with nonequilibrium processes. To theoretically describe a system that experiences nonequilibrium situations, one powerful approach is the Keldysh technique within the Green function framework^{50,51}. In this section, we start with the Hamiltonian of surface states of a three dimensional topological insulator and generalize the Keldysh technique to a Josephson structure made of the disordered surface states of a three dimensional TI under nonequilibrium. The system under consideration is schematically shown in Fig. 1(a).

The Hamiltonian that describes the Rashba type surface states in the presence of an in-plane exchange field $\mathbf{h}(\mathbf{r}) = (h_x(\mathbf{r}), h_y(\mathbf{r}), 0)$ reads:

$$\hat{H}(\mathbf{r}) = -i\alpha(\nabla \times \mathbf{e}_z)\hat{\sigma} + \mathbf{h}(\mathbf{r})\hat{\sigma} + V_{imp}(\mathbf{r}) - \mu, \quad (1)$$

$$\hat{H} = \int d^2\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}') \hat{H}(\mathbf{r}') \hat{\Psi}(\mathbf{r}'), \quad (2)$$

in which $\hat{\Psi} = (\Psi_\uparrow, \Psi_\downarrow)^T$, α is the Fermi velocity, \mathbf{e}_z is a unit vector normal to the surface of TI (see Fig. 1(a)), μ is the chemical potential, and $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector of Pauli matrices in the spin space. Note that the Dirac type of Hamiltonian would only change the notations¹². We assume that the system involves nonmagnetic impurities that can be described by a Gaussian scattering potential: $V_{imp}(\mathbf{r}) = \sum_{\mathbf{r}_i} V_i \delta(\mathbf{r} - \mathbf{r}_i)$

$$\langle V(\mathbf{r})V(\mathbf{r}') \rangle = \frac{1}{\pi\nu\tau} \delta(\mathbf{r} - \mathbf{r}'), \quad \nu = \frac{\mu}{2\pi\alpha^2}. \quad (3)$$

Here τ is the mean free time of quasiparticles and ν is the density of states at the Fermi level of the normal state of TI.

The advanced (A), retarded (R), and Keldysh (K) blocks of Gor'kov Green function in the Keldysh technique are defined as follows:

$$G_{\alpha\beta}^{R,A}(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = \mp i \Theta_{t_1,2} \langle \{ \Psi_\alpha(\mathbf{r}_1, t_1), \Psi_\beta^\dagger(\mathbf{r}_2, t_2) \} \rangle, \quad (4a)$$

$$F_{\alpha\beta}^{R,A}(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = \pm i \Theta_{t_1,2} \langle \{ \Psi_\alpha(\mathbf{r}_1, t_1), \Psi_\beta(\mathbf{r}_2, t_2) \} \rangle, \quad (4b)$$

$$F_{\alpha\beta}^{\dagger R,A}(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = \pm i \Theta_{t_1,2} \langle \{ \Psi_\alpha^\dagger(\mathbf{r}_1, t_1), \Psi_\beta^\dagger(\mathbf{r}_2, t_2) \} \rangle, \quad (4c)$$

$$\tilde{G}_{\alpha\beta}^{R,A}(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = \mp i \Theta_{t_1,2} \langle \{ \Psi_\alpha^\dagger(\mathbf{r}_1, t_1), \Psi_\beta(\mathbf{r}_2, t_2) \} \rangle, \quad (4d)$$

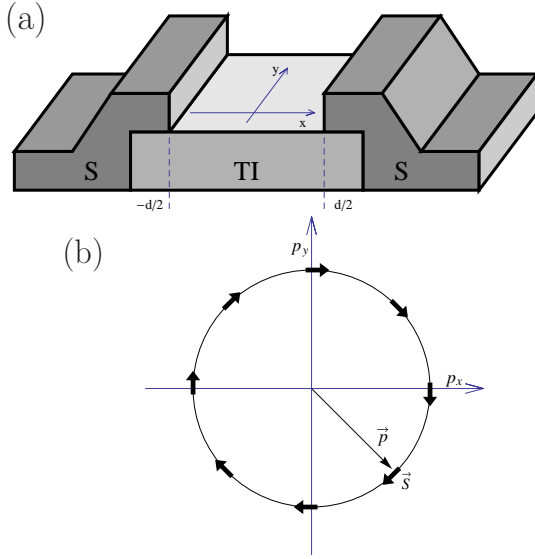


FIG. 1. (a) Schematic of the two dimensional S-TI-S hybrid junction. The surface states of a three dimensional topological insulator is sandwiched between two s wave superconductors. The junction length along the x direction is equal to d while its width along y is assumed enough large so that the influences of boundaries in this direction are negligible. The junction plane resides in the xy plane and S-TI interfaces are located at $x = \pm d/2$. (b) The spin-momentum locking phenomenon in the conduction band of surface channels.

$$G_{\alpha\beta}^K(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = -i\langle [\Psi_\alpha(\mathbf{r}_1, t_1), \Psi_\beta^\dagger(\mathbf{r}_2, t_2)] \rangle, \quad (4e)$$

where $\{\dots, \dots\}$ and $[\dots, \dots]$ stand for anticommutator and commutator relations, respectively. Also, the time ordering operator is shown by a step function in time $\Theta_{t_1,2} \equiv \Theta(t_1 - t_2)$. The other components of Keldysh block i.e. $F_{\alpha\beta}^K$, $F_{\alpha\beta}^{\dagger K}$, and $\tilde{G}_{\alpha\beta}^K$ are related to this component Eq. (4e) the same way as those of the retarded and advanced components given above, namely Eqs. (4b)-(4d), to Eq. (4a).

We now introduce the following matrix Green function in the Nambu space:

$$\tilde{G}^R = \begin{pmatrix} \hat{G}_{new}^R & \hat{F}_{new}^R \\ \hat{F}_{new}^{\dagger R} & \hat{G}_{new}^R \end{pmatrix} = \begin{pmatrix} \hat{G}_{old}^R & \hat{F}_{old}^R i\sigma_y \\ i\sigma_y \hat{F}_{old}^{\dagger R} & -\sigma_y \hat{G}_{old}^R \sigma_y \end{pmatrix}, \quad (5)$$

where \hat{G}_{old}^R (\hat{F}_{old}^R , $\hat{F}_{old}^{\dagger R}$, \hat{G}_{old}^R) are 2×2 matrices in the spin space. The elements of these matrices are given above by Eqs. (4a)-(4d). This definition can be extended to the advanced and keldysh blocks \tilde{G}^A and \tilde{G}^K . It is also convenient to introduce a full 8×8 Green function in the Keldysh space as follows:

$$\tilde{G} = \begin{pmatrix} \tilde{G}^R & \tilde{G}^K \\ 0 & \tilde{G}^A \end{pmatrix}. \quad (6)$$

Therefore, by averaging the Green function over the impurity scattering potential in the Born approximation we

find the following Gor'kov equation:

$$\begin{pmatrix} i\partial_{t_1} - \hat{H}(\mathbf{r}_1) & 0 \\ 0 & -i\partial_{t_1} - \sigma_y \hat{H}^*(\mathbf{r}_1) \sigma_y \end{pmatrix} \tilde{G} = \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(t_1 - t_2) + \frac{1}{\pi\nu\tau} \tilde{G}(\mathbf{r}_1, \mathbf{r}_1) \tilde{G}(\mathbf{r}_1, \mathbf{r}_2). \quad (7)$$

Here, the two time dependent products of AB operators is equivalent to $AB(t_1, t_2) \equiv \int dt' A(t_1, t') B(t', t_2)$.

We consider a situation where the chemical potential μ is the largest energy scale in the system and hence a quasiclassical approximation is the well suited framework to describe the system. We now can introduce the quasiclassical Green function

$$\tilde{g} = \frac{i}{\pi} \int d\xi \tilde{G} = \begin{pmatrix} \hat{g} & \hat{f} \\ \hat{f} & \hat{g} \end{pmatrix}, \quad (8)$$

where a Fourier transformation of the Green function with respect to the relative space arguments and an integration over $\xi = \alpha p - \mu$ are performed. Furthermore, following the standard procedures¹², we find the Eilenberger equation for \tilde{g} :

$$\frac{\alpha}{2} \{ \hat{\eta}, \nabla \tilde{g} \} = -\hat{\tau}_z \partial_{t_1} \tilde{g} - \partial_{t_2} \tilde{g} \hat{\tau}_z + \left[\tilde{g}, i\mathbf{h}(\mathbf{R}) \hat{\sigma} \hat{\tau}_z + i\mu \hat{\eta} \mathbf{n}_F + \frac{\langle \tilde{g} \rangle}{\tau} \right], \quad (9)$$

in which $\hat{\eta} = (-\sigma_y, \sigma_x, 0)$, $\mathbf{n}_F = \mathbf{p}_F / |\mathbf{p}_F|$, and τ_i , $i = x, y, z$ are Pauli matrices in the particle-hole space. This is a nonequilibrium generalization to the Eilenberger equation derived in Ref. 12 for the equilibrium situations. Recently, the analogous quasiclassical equation was also discussed for Dirac edge and surface electrons with another type of spin-momentum locking $\mathbf{p}_F \sigma^5$. In the stationary situations, that we consider throughout the paper, one can perform a Fourier transformation with respect to $t_1 - t_2 \rightarrow \varepsilon$. Thus, the Eilenberger equation can be expressed by:

$$\frac{\alpha}{2} \{ \hat{\eta}, \nabla \tilde{g} \} = \left[\tilde{g}, -i\varepsilon \hat{\tau}_z + i\mathbf{h}(\mathbf{R}) \hat{\sigma} \hat{\tau}_z + i\mu \hat{\eta} \mathbf{n}_F + \frac{\langle \tilde{g} \rangle}{\tau} \right]. \quad (10)$$

To find a general solution to Eq. (10), the Green function can be expanded by the Pauli matrices as follows;

$$\tilde{g}(\varepsilon, \mathbf{R}, \mathbf{n}_F) = \frac{1}{2} \hat{g}' + \frac{1}{2} \hat{g}'' \hat{\eta} \mathbf{n}_F + \hat{g}_{||} \mathbf{n}_F \hat{\sigma} + \hat{g}_z \sigma_z, \quad (11)$$

where \hat{g}' , \hat{g}'' , $\hat{g}_{||}$, and \hat{g}_z depend on $(\varepsilon, \mathbf{R}, \mathbf{n}_F)$ and are 4×4 matrices in the particle-hole and Keldysh space. It can be shown that $\hat{g}_{||}$ and \hat{g}_z are smaller than both \hat{g}' and \hat{g}'' by a factor of order of $(\tau^{-1}, h, \varepsilon) / \mu \ll 1$. Therefore, one can safely neglect these terms in the expansion without missing significant information. The Eilenberger equation in this approximation decomposes into two coupled equations for \hat{g}' and \hat{g}'' :

$$\alpha \mathbf{n}_F \nabla \hat{g}'' = \left[\hat{g}', -i\varepsilon \hat{\tau}_z + \frac{\langle \hat{g}' \rangle}{2\tau} \right] + \left[\hat{g}'', i\mathbf{h}_1 \hat{\tau}_z + \frac{\langle \hat{g}'' \rangle}{2\tau} \right], \quad (12a)$$

$$\alpha \mathbf{n}_F \nabla \hat{g}' = \left[\hat{g}'', -i\varepsilon \hat{\tau}_z + \frac{\langle \hat{g}' \rangle}{2\tau} \right] + \left[\hat{g}', ih_1 \hat{\tau}_z + \frac{\langle \hat{g}'' \rangle}{2\tau} \right], \quad (12b)$$

where $h_1 = h_x n_{F,y} - h_y n_{F,x}$. It is apparent that Eqs. (12a) and (12b) allow for a solution of $\hat{g}' = \hat{g}''$. This means that the spin structure of full Green function is determined by $(1 + \hat{\eta} \mathbf{n}_F)/2$. Physically, this operator projects the Green function onto the conduction band of TI surface states as schematically depicted in Fig. 1(b). We assume that this band can describe the surface electrons of TI and take the spinless Green function \hat{g}' in the rest of our calculations. Now, equations (12a) and (12b) reduce to a single spinless equation so that \hat{g}'^2 satisfies an equation of the same form. Also, the normalization condition i.e. $\hat{g}'^2 = 1$ is a solution to this equation.

In the diffusive regime, where the quasiparticles' motion is fully randomized by strong scattering resources so that $\varepsilon, h < \tau^{-1}$, the Eilenberger equation reduces to the Usadel equation⁵³. To have a self-contained presentation, we here give a short recap of basic notations and equations generalized for a nonequilibrium situation¹². In the diffusive regime, the Green function can be expanded through the first two harmonics:

$$\hat{g}'(\varepsilon, \mathbf{R}, \mathbf{n}_F) = \hat{g}_s(\varepsilon, \mathbf{R}) + \mathbf{n}_F \hat{g}_a(\varepsilon, \mathbf{R}), \quad (13)$$

where the zeroth harmonic (isotropic) is larger than the first harmonic: $\hat{g}_s \gg \hat{g}_a$. Following the derivation steps described in Ref. 12, one obtains the Usadel equation to \hat{g}_s :

$$D \hat{\nabla}(\hat{g}_s \hat{\nabla} \hat{g}_s) = [-i\varepsilon \hat{\tau}_z, \hat{g}_s], \quad (14)$$

where $D = \alpha^2 \tau$ is the diffusion constant and the first harmonic term can be expressed in terms of \hat{g}_s as follows:

$$\hat{g}_a = -2\alpha\tau(\hat{g}_s \hat{\nabla} \hat{g}_s). \quad (15)$$

The operator $\hat{\nabla}$ is defined for the Rashba type band structure of surface states as follows:

$$\hat{\nabla} X = \nabla X + \frac{i}{\alpha} (h_x \mathbf{e}_y - h_y \mathbf{e}_x) [\tau_z, X]. \quad (16)$$

Note that the formalism can be straightforwardly extended to the Dresselhaus type¹². In the junction configuration systems, the Usadel equation should be supplied by appropriate boundary conditions. Here, we consider low transparent tunnelling processes at the TI-S interfaces shown in Figs. 1(a) and 2. This boundary condition is experimentally relevant and was discussed in Refs. 54 and 55 for conventional metallic junctions. We assume that the s wave superconducting terminals with a gap of Δ in their energy spectrums are in the equilibrium state and thus can be described by the bulk solution \hat{g}_{SC} at the boundaries¹²:

$$2\gamma \hat{g}_s \mathbf{n} \hat{\nabla} \hat{g}_s = [\hat{g}_s, \hat{g}_{SC}], \quad (17)$$

where γ is the ratio of resistance of the interface barrier per unit area to the resistivity of the TI surface states and

\mathbf{n} is the unit vector normal to the interface (it points towards the superconductor region). In the tunneling low proximity regime we consider throughout our calculations, the opacity of interfaces γ should be sufficiently large to satisfy this condition. The superconducting bulk solution can be given by:

$$\hat{g}_{SC}^{R,A} = \pm \frac{\text{sgn}\varepsilon}{\sqrt{(\varepsilon \pm i\delta)^2 - \Delta^2}} [\varepsilon \tau_z + \Delta \tau_+ - \Delta^* \tau_-] \\ \hat{g}_{SC}^K = (\hat{g}_{SC}^R - \hat{g}_{SC}^A) \tanh \frac{\varepsilon}{2T}, \quad (18)$$

where $\tau_{\pm} = (\tau_x \pm i\tau_y)/2$, δ is an infinitesimal positive value and the system temperature is denoted by T .

In the context of quantum transport, the electric current is one of the most important physical quantities that can explain transport experiments. The electric current density across the surface channels of TI junction we consider can be expressed by:

$$\mathbf{j} = -\frac{ie\alpha}{4} \lim_{\mathbf{r} \rightarrow \mathbf{r}'} \text{Tr}_4 \int \frac{d\varepsilon}{2\pi} \hat{\eta} \hat{\tau}_z \check{G}^K(\mathbf{r}, \mathbf{r}', \varepsilon). \quad (19)$$

Rewriting the current density via the quasiclassical Green functions in the diffusive limit we obtain:

$$\mathbf{j} = \frac{\sigma_N}{8e} \text{Tr}_2 \int d\varepsilon \left[\hat{\tau}_z (\hat{g}_s \hat{\nabla} \hat{g}_s)^K \right], \quad (20)$$

where $\sigma_N = 2e^2 \nu D$ is the normal state conductivity.

III. MAGNETOELECTRIC EFFECT

In this section, we develop a theory of the dissipationless direct magnetoelectric effect to the disordered topological insulator Josephson junction depicted in Fig. 1(a). As it was mentioned earlier in the introduction, in response to a dc Josephson current flowing across the TI junction, a stationary spin density S_y oriented along the junction interface can be generated. To uncover this phenomenon, we first evaluate the average spin polarization in TI which is:

$$\mathbf{S} = \frac{1}{2} \langle \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\sigma} \hat{\Psi}(\mathbf{r}, t) \rangle. \quad (21)$$

In terms of the Green function, the components of spin polarization take the following form:

$$S_i = -\frac{i}{8} \text{Tr}_4 \int \frac{d\varepsilon}{2\pi} \frac{d^2 p}{(2\pi)^2} \hat{\sigma}_i \hat{\tau}_z \check{G}^K(\mathbf{p}, \mathbf{R}, \varepsilon) = \\ -\frac{\nu}{16} \text{Tr}_4 \int d\varepsilon \hat{\sigma}_i \hat{\tau}_z \langle \check{g}^K \rangle. \quad (22)$$

Within the quasiclassical diffusive regime we consider for the disordered topological insulator, the spin polarization components can be rewritten as

$$S_i = -\frac{\nu}{16} \text{Tr}_4 \int d\varepsilon \hat{\sigma}_i \hat{\tau}_z \left\langle \frac{1 + \hat{\eta} \mathbf{n}_F}{2} (\hat{g}_s + \hat{g}_a \mathbf{n}_F)^K \right\rangle \\ = -\frac{\nu}{64} \text{Tr}_4 \int d\varepsilon [\hat{\sigma}_i \hat{\tau}_z \hat{g}_a^K \hat{\eta}]. \quad (23)$$

Substituting $\hat{\mathbf{g}}_a^K$ from Eq. (15) into Eq. (23), we obtain:

$$S_{x,y} = \pm \frac{\nu}{32} \text{Tr}_2 \int d\varepsilon [\hat{\tau}_z \hat{\mathbf{g}}_a^K \mathbf{e}_{y,x}] = \mp \frac{\nu}{16} \alpha \tau \text{Tr}_2 \int d\varepsilon [\hat{\tau}_z \hat{g}_s \hat{\partial}_{y,x} \hat{g}_s]^K. \quad (24)$$

Comparing the last expression to Eq. (20), we see that

$$S_{x,y} = \mp \frac{1}{4e\alpha} j_{y,x}. \quad (25)$$

That is, the electric current and electron spin polarization orientation are perpendicular while their amplitudes are directly proportional.

When a superconducting phase gradient ϕ is applied between the superconducting terminals, a dc Josephson current flows across the junction. In the tunneling and low proximity limit, the supercurrent has a standard sinusoidal relation $I = I_c \sin \phi$ in which I_c is the critical supercurrent¹². Therefore, considering the spin polarization relation given above, we see that the spin polarization can be simply controlled by the superconducting phase difference ϕ . This is a generic phenomenon and occurs in ballistic systems as well. To derive an expression for the spin density in the ballistic regime, we consider a surface channel with superconductivity where the superconducting phase ϕ experiences a coordinate gradient so that $\nabla \phi(\mathbf{r}) \neq 0$. The spin density in momentum representation and in the limit when the superconducting gap is much smaller than the temperature is given by

$$\mathbf{S}(\mathbf{q}) = -\frac{1}{2} \text{Tr}_2 T \sum_{\omega_n} \int \frac{d\mathbf{p} d\mathbf{q}'}{(2\pi)^4} \hat{\boldsymbol{\sigma}} \left[\hat{G}(i\omega_n, \mathbf{p} + \frac{\mathbf{q}}{2}) \hat{\Delta}(\mathbf{q}' + \frac{\mathbf{q}}{2}) \times \hat{G}^\dagger(-i\omega_n, -\mathbf{p} + \mathbf{q}') \hat{\Delta}^\dagger(\mathbf{q}' - \frac{\mathbf{q}}{2}) \hat{G}(i\omega_n, \mathbf{p} - \frac{\mathbf{q}'}{2}) \right], \quad (26)$$

where $\hat{\Delta}(\mathbf{q}) = i\hat{\sigma} \Delta(\mathbf{q})$. The Green function in the Matsubara representation reads:

$$\hat{G}(i\omega_n, \mathbf{p}) = \left[i\omega_n - \hat{H}(\mathbf{p}) \right]^{-1} = \frac{1}{2} \sum_{s=\pm 1} \frac{1 + s[\mathbf{n}_F \times \mathbf{e}_z] \hat{\boldsymbol{\sigma}}}{i\omega_n + \mu - s\alpha p}. \quad (27)$$

Following similar steps as Ref. 29, one obtains the spin density in the real space representation as:

$$\mathbf{S}(\mathbf{r}) = -i \text{Tr}_2 \frac{\hat{\boldsymbol{\sigma}}(\hat{\mathbf{q}} \cdot \mathbf{F}(\mathbf{r}))}{16\pi\alpha} T \sum_{\omega_n > 0} \left[1 + \frac{\mu}{\omega_n} \text{atan} \frac{\mu}{\omega_n} + \frac{\omega_n}{2\mu} \left\{ \frac{\pi}{2} - \text{atan} \left[\frac{\mu}{2\omega_n} \left(1 - \frac{\omega_n^2}{\mu^2} \right) \right] \right\} \right] \frac{1}{\omega_n^2}, \quad (28)$$

where $\mathbf{F}(\mathbf{r}) = (\nabla \Delta^*(\mathbf{r})) \Delta(\mathbf{r}) - \Delta^*(\mathbf{r}) (\nabla \Delta(\mathbf{r}))$. Considering $\Delta(\mathbf{r}) = |\Delta| e^{i\phi(\mathbf{r})}$ one finds that at the charge neutrality point at which $\mu = 0$ the spin density reduces to:

$$S_{x,y}(\mathbf{r}) = \mp \frac{|\Delta|}{16\pi\alpha} \frac{|\Delta|}{T} \partial_{y,x} \phi(\mathbf{r}), \quad (29)$$

while within $\mu \gg T_c$ regime one arrives at:

$$S_{x,y}(\mathbf{r}) = \mp \alpha \nu \frac{7\zeta(3)}{(8\pi)^2} \left(\frac{|\Delta|}{T} \right)^2 \partial_{y,x} \phi(\mathbf{r}). \quad (30)$$

As seen, in both regimes, the spin polarization and the direction of the current (which is proportional to the phase gradient $\nabla \phi(\mathbf{r})$) are orthogonal to each other.

We note that similar direct magnetoelectric effect was also predicted for superconductors and superconducting heterostructures with intrinsic spin orbit interactions²⁹⁻³¹. In these systems, a Rashba spin orbit interaction is considered to be present and a generic relation to the spin polarization obtained $S_y = \chi(j_x/ev_F)$. Nonetheless, the coefficient χ in realistic systems of this type is practically negligible and proportional to $\sim \Delta_{so}/\varepsilon_F$, where Δ_{so} is the splitting of the conducting bands due to the spin orbit interaction and ε_F is the Fermi energy.

This issue however can be resolved in the surface channels of a 3D TI. The surface electrons are fully spin-momentum locked and therefore they have only one conduction band. Hence, the relation between the spin polarization and supercurrent contains no reducing coefficient of order of $\Delta_{so}/\varepsilon_F$ and the effect should be much stronger than that of the conventional spin orbit coupled materials with $\Delta_{so}/\varepsilon_F \ll 1$ discussed so far in the literature.

IV. INVERSE MAGNETOELECTRIC EFFECT

In this section, we first show that a nonequilibrium quasiparticle injection on the surface of 3D TI induces a Zeeman-like field which is proportional to the voltage difference between an injector electrode and superconducting terminals that drives the quasiparticle flow. We then calculate the dc Josephson current through a disordered topological insulator junction where a flow of nonequilibrium quasiparticle current is injected into the TI region. We discuss the appearance of ϕ_0 states which is controllable by means of the quasiparticle flow injection.

A. Zeeman-like field induced by quasiparticle injection

In this subsection, we demonstrate that an electric current flowing through the surface states of the 3D TI can induce a Zeeman-like term. We consider a highly simple model which is sufficient to reveal this effect. In order to more simplify our discussions, we consider fully ballistic surface states without any disorder. In this case, the Hamiltonian is given by Eq. (1) where $V_{imp}(\mathbf{r}) = \mathbf{h}(\mathbf{r}) = 0$. Since we have employed the quasichlassical approximation where $\mu \gg \varepsilon$, we only consider the conduction band and assume in Eq. 27 that

$$\hat{G}(i\omega_n, \mathbf{p}) = \frac{1 + [\mathbf{n}_F \times \mathbf{e}_z] \hat{\boldsymbol{\sigma}}}{2(i\omega_n + \mu - \alpha p)}. \quad (31)$$

Moreover, we include the electron-electron interaction at the Hartree-Fock level. Thus, the self-energy reads:

$$\hat{\Sigma} = \hat{\Sigma}^{(1)} + \hat{\Sigma}^{(2)}, \quad (32)$$

where

$$\hat{\Sigma}^{(1)} = \text{Tr}_2 T \sum_n \int \frac{d^2 p}{(2\pi)^2} V(0, 0) \hat{G}(i\omega_n, \mathbf{p}), \quad (33a)$$

$$\hat{\Sigma}^{(2)} = -T \sum_n \int \frac{d^2 p_1}{(2\pi)^2} V(\mathbf{p}, \mathbf{p}_1) \hat{G}(i\omega_n, \mathbf{p}_1). \quad (33b)$$

We also assume that μ contains the spin independent part of self-energy and focus on the spin part of $\hat{\Sigma} = \mathbf{\Sigma} \hat{\sigma}$. The Green function with interactions reads:

$$\hat{G}(i\omega_n, \mathbf{p}) = \left[i\omega_n + \mu - \alpha \hat{\sigma} [\mathbf{p} \times \mathbf{e}_z] - \mathbf{\Sigma} \hat{\sigma} \right]^{-1} = \frac{1 + (\alpha \hat{\sigma} [\mathbf{p} \times \mathbf{e}_z] + \mathbf{\Sigma} \hat{\sigma})/\varepsilon_0}{2(i\omega_n + \mu - \varepsilon_0)}, \quad (34)$$

where $\mathbf{\Sigma} = (\Sigma_x, \Sigma_y, \Sigma_z)$ and

$$\varepsilon_0 = \sqrt{(\alpha p_y + \Sigma_x)^2 + (-\alpha p_x + \Sigma_y)^2 + \Sigma_z^2}. \quad (35)$$

By now substituting the Green function Eq. (34) into Eqs. (33a) and (33b) and assuming that $V(\mathbf{p}, \mathbf{p}_1) = \lambda$ we find that the spin dependent part of the self-energy is:

$$\mathbf{\Sigma} = \frac{\lambda}{4} \int \frac{d\mathbf{p}}{(2\pi)^2} \frac{\alpha [\mathbf{p} \times \mathbf{e}_z] + \mathbf{\Sigma}}{\varepsilon_0} \tanh \frac{\varepsilon_0 - \mu}{2T}. \quad (36)$$

In equilibrium at $V = 0$, the spin dependent part of the self-energy vanishes due to the integration over momentum. However, in the presence of bias voltage the integral results in a nonvanishing finite value. If we assume an electric current in the x direction of a ballistic system, we then are able to substitute $\mu \rightarrow \mu + (V/2)\text{sgn}p_x$ in Eq. (36). In the linear approximation with respect to the dimensionless interaction constant $\lambda\nu$ together with $|\Sigma| \ll \mu$ we find:

$$\mathbf{\Sigma} = -\frac{\lambda}{4} \int \frac{d\mathbf{p}}{(2\pi)^2} V \text{sgn}p_x \delta(\alpha p - \mu) [\mathbf{n} \times \mathbf{e}_z], \quad (37)$$

where we set $T = 0$ and expand Eq. (36) around V to reach at a liner response theory. Hence, the only nonzero

component is Σ_y :

$$\Sigma_y = \frac{\lambda\nu V}{2\pi}. \quad (38)$$

In conclusion, based on a simplified model, we have shown that an electric current can generate spin dependent exchange that results in an effective Zeeman-like term. This induced exchange field can be controlled by the applied voltage to inject quasiparticles. A more detailed study of the particular connection between the induced Zeeman-like term and the injected quasiparticles' current is an interesting problem, however, is beyond the scope of the present paper and can be addressed elsewhere.

B. Josephson current through a 3D TI under quasiparticle flow injection

The flow injection of quasiparticles into the junction generates a nonequilibrium situation in the TI region. The Josephson current in this nonequilibrium situation is determined not only by the condensate wave functions in the TI region but also by the nonequilibrium distribution function in this segment. The current can be calculated by Eq. (20). It is convenient to calculate it at the interfaces of TI-S by exploiting the boundary conditions Eq. (17):

$$j_x = \pm \frac{\sigma_N}{8e} \text{Tr}_2 \int d\varepsilon \frac{1}{2\gamma} \hat{\tau}_z [\hat{g}_s, \hat{g}_{SC}]^K, \quad (39)$$

where \pm signs refer to the boundary conditions at $x = \pm d/2$. The Keldysh component of the full Green function can be expressed via the retarded and advanced components with the distribution function:

$$\hat{g}_s^K = \hat{g}_s^R \hat{\varphi} - \hat{\varphi} \hat{g}_s^A = \begin{pmatrix} g_s^R \varphi - \varphi g_s^A & f_s^R \tilde{\varphi} - \varphi f_s^A \\ \tilde{f}_s^R \varphi - \tilde{\varphi} \tilde{f}_s^A & \tilde{g}_s^R \tilde{\varphi} - \tilde{\varphi} \tilde{g}_s^A \end{pmatrix}, \quad (40)$$

where the distribution function is a diagonal matrix in the particle-hole space:

$$\hat{\varphi} = \begin{pmatrix} \varphi & 0 \\ 0 & \tilde{\varphi} \end{pmatrix}. \quad (41)$$

There is a general relation (due to the particle-hole symmetry) between φ and $\tilde{\varphi}$. For our case of spinless fermions it reduces to $\tilde{\varphi}(\varepsilon) = -\varphi(-\varepsilon)$.

Utilizing the distribution function, the current Eq. (39) can be expressed by:

$$j_x = \pm \frac{\sigma_N}{8e\gamma} \int d\varepsilon \left\{ -2 [\text{Re} g_{SC}^R] (\varphi - \tilde{\varphi}) - [\text{Re} f_{SC}^R] \tanh \frac{\varepsilon}{2T} (\tilde{f}_s^R e^{\pm i\phi/2} + f_s^R e^{\mp i\phi/2} + f_s^A e^{\mp i\phi/2} + \tilde{f}_s^A e^{\pm i\phi/2}) + [i \text{Im} f_{SC}^R] \left((e^{\pm i\phi/2} \tilde{f}_s^A - e^{\mp i\phi/2} f_s^R) \tilde{\varphi} + (e^{\mp i\phi/2} f_s^A - e^{\pm i\phi/2} \tilde{f}_s^R) \varphi \right) \right\}. \quad (42)$$

Here, we assume that the left and right superconducting

terminals possess $\mp\phi/2$ macroscopic phases so that the

phase difference between the superconductors is ϕ . The anomalous components of Green function at the interfaces are denoted by $f_s^{R,A}$ and $\tilde{f}_s^{R,A}$. In order to find the Josephson current to the leading order with respect to the interface conductance γ^{-1} we expand the Green function around the bulk solution in the TI region. To obtain Eq. (42) we have already taken into account this regime and set $g^{R,A} = \pm 1$ to the first order in γ^{-1} .

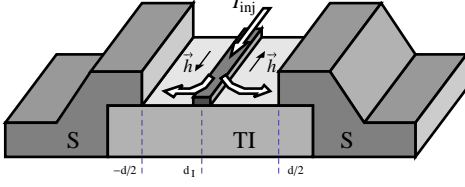


FIG. 2. Schematic of the topological insulator Josephson junction proposed to generate a ϕ_0 -junction by means of quasiparticle flow injection. The junction resides in the xy plane (the same as Fig. 1(a)) and the injector electrode is located at $x = d_I$ while the superconducting interfaces are assumed at $x = \pm d/2$. The quasiparticle flow is controlled by a voltage difference between the superconducting terminals and injector electrode.

The experimental setup we propose with the electronic current injection is schematically shown in Fig. 2. The setup is made of two superconductor terminals connected via the surface channels of a 3D TI at $x = \pm d/2$. The junction plane resides in the xy plane and a normal electrode that injects quasiparticles into the junction is attached at $x = d_I$. The quasiparticles flow can be induced by applying a voltage difference between the superconducting terminals and normal injector electrode. We assume that the voltage difference between the two superconducting terminals is zero so that we are able to eliminate the ambiguities and complications concerning nonequilibrium phenomena that arise when the two superconductors possess a finite voltage difference. Therefore, the quasiparticles current has opposite directions within each region of the TI surface separated by the injector electrode at $x = d_I$. The direction of quasiparticles current is shown by white arrows in Fig. 2 and we assume that the current is uniform along the y direction parallel to the S-TI interfaces. As discussed in passing, the electric current within TI in the $x(y)$ direction generates a finite spin polarization along the $y(x)$ direction. Therefore, as it was shown in the previous subsection, this electric current injection induces an effective Zeeman-like energy $h \propto V$ in the TI region where V is the voltage difference applied between the normal injector electrode and the superconductors. The resulting exchange fields \vec{h} are shown in Fig. 2. The proposed setup generates two domains with opposite orientations for the effective Zeeman-like term which are locked to the direction of injected electric currents. This effective

Zeeman-like domain scenario can be described by:

$$h_y(x) = \begin{cases} -h & , -d/2 < x < d_I \\ +h & , d_I < x < +d/2 \end{cases} . \quad (43)$$

To find the anomalous Green function within the TI region, we solve the linearized Usadel equation in the surface channels:

$$D\kappa(\partial_x - \frac{2i}{\alpha}h_y(x))^2 f_s^{R,A} = -2i\varepsilon f_s^{R,A}, \quad (44a)$$

$$D\kappa(\partial_x + \frac{2i}{\alpha}h_y(x))^2 \tilde{f}_s^{R,A} = -2i\varepsilon \tilde{f}_s^{R,A}, \quad (44b)$$

where $\kappa = \pm 1$ refer to the retarded and advanced components.

The Usadel equation should be solved together with the boundary conditions Eq. (17) at S-TI interfaces:

$$(\partial_x - \frac{2i}{\alpha}h_y)f_s^{R,A}|_{x=\mp d/2} = \mp \frac{1}{\gamma}f_{SC}^{R,A}e^{\mp i\phi/2}, \quad (45a)$$

$$(\partial_x + \frac{2i}{\alpha}h_y)\tilde{f}_s^{R,A}|_{x=\mp d/2} = \pm \frac{1}{\gamma}f_{SC}^{R,A}e^{\pm i\phi/2}, \quad (45b)$$

and at the interface between the domains ($x = d_I$):

$$f_s^{R,A}|_{x=d_I-\epsilon} = f_s^{R,A}|_{x=d_I+\epsilon}, \quad (46a)$$

$$(\partial_x - \frac{2i}{\alpha}h_y)f_s^{R,A}|_{x=d_I-\epsilon} = (\partial_x - \frac{2i}{\alpha}h_y)f_s^{R,A}|_{x=d_I+\epsilon}, \quad (46b)$$

$$\tilde{f}_s^{R,A}|_{x=d_I-\epsilon} = \tilde{f}_s^{R,A}|_{x=d_I+\epsilon}, \quad (46c)$$

$$(\partial_x + \frac{2i}{\alpha}h_y)\tilde{f}_s^{R,A}|_{x=d_I-\epsilon} = (\partial_x + \frac{2i}{\alpha}h_y)\tilde{f}_s^{R,A}|_{x=d_I+\epsilon}, \quad (46d)$$

where $\epsilon \rightarrow 0$. The solution to $f_s^{R,A}$ component takes the following form:

$$f_s^{R,A}(x) = \frac{f_{SC}^{R,A}e^{\frac{2i}{\alpha}h_y(x)(x-d_I)}}{\gamma\lambda^{R,A}\sinh[\lambda^{R,A}d]} \left\{ e^{-i\frac{\phi}{2}-i\chi_l} \times \cosh[\lambda^{R,A}(x - \frac{d}{2})] + e^{i\frac{\phi}{2}-i\chi_r} \cosh[\lambda^{R,A}(x + \frac{d}{2})] \right\}, \quad (47)$$

in which $\lambda^{R,A} = \sqrt{-2i\kappa\varepsilon/D}$ and $\chi_{l,r} = 2h(d/2 \pm d_I)/\alpha$. To calculate the current across the junction, one needs also to obtain a solution to $\tilde{f}_s^{R,A}$. The solution to this component however can be simply given as $\tilde{f}_s^{R,A} = -f_s^{R,A}(\phi \rightarrow -\phi, h_y(x) \rightarrow -h_y(x))$. By substituting the obtained $f_s^{R,A}$ and $\tilde{f}_s^{R,A}$ into the current definition (42), we are able to analyze the transport properties of the system proposed. The current relation (42) can be decomposed to two components

$$j_x = j_1 + j_2, \quad (48)$$

where

$$j_1 = \pm \frac{\sigma_N}{8e\gamma} \int d\varepsilon \{ -2 [\text{Reg}_{SC}^R] (\varphi - \bar{\varphi}) \}, \quad (49)$$

and j_2 is expressed by the remaining parts of Eq. (42) that contains the anomalous Green functions. Note that because of opposite signs at the S-TI interfaces, the contribution of j_1 component to the Josephson current vanishes. Physically, this component represents a part of

the injected quasiparticle current. Specifically, it can be seen that j_1 is equal to zero in the equilibrium simply because $\varphi(\varepsilon) = \tilde{\varphi}(\varepsilon) = \tanh \varepsilon/2T$. Therefore, j_2 component contains the Josephson current and we focus on this component of the current in the following. Since the anomalous components of Green function are of the first order in γ^{-1} , we thus only need the distribution function in the zeroth order with respect to this parameter for calculating j_2 . In this regime we have

$$\varphi(\varepsilon) = \tanh \frac{\varepsilon - V}{2T}, \quad (50a)$$

$$\tilde{\varphi}(\varepsilon) = -\varphi(-\varepsilon) = \tanh \frac{\varepsilon + V}{2T}. \quad (50b)$$

These expressions imply that the main contribution to the resistance of the system originates from the tunnel process at the TI-S interfaces. The following combination of the distribution functions enables us to even decompose j_2 component of current to two parts $j_{2,1}$ and $j_{2,2}$:

$$\varphi_{\pm}(\varepsilon) = \varphi(\varepsilon) \pm \tilde{\varphi}(\varepsilon) = \tanh \frac{\varepsilon - V}{2T} \pm \tanh \frac{\varepsilon + V}{2T}, \quad (51)$$

$$j_{2,1} = \mp \frac{\sigma_N}{2e\gamma^2} \int d\varepsilon [\text{Im} f_{SC}^R] \frac{\varphi_{\pm}}{2} \left(\cos(\phi - \chi) \text{Im} \left[\frac{f_{SC}^R}{\lambda^R \sinh[\lambda^R d]} \right] + \text{Im} \left[\frac{f_{SC}^R}{\lambda^R \tanh[\lambda^R d]} \right] \right), \quad (52a)$$

$$j_{2,2} = -\frac{\sigma_N}{2e\gamma^2} \sin(\phi - \chi) \int d\varepsilon \left\{ [\text{Re} f_{SC}^R] \tanh \frac{\varepsilon}{2T} \text{Im} \left[\frac{f_{SC}^R}{\lambda^R \sinh[\lambda^R d]} \right] + [\text{Im} f_{SC}^R] \frac{\varphi_{\pm}}{2} \text{Re} \left[\frac{f_{SC}^R}{\lambda^R \sinh[\lambda^R d]} \right] \right\}, \quad (52b)$$

in which $\chi = \chi_r - \chi_l$ and \pm at the right hand side of Eq. (52a) pertinent to the left and right interfaces. As seen, $j_{2,1}$ reverses its direction at the left and right S-TI interfaces. Consequently, this component is unable to contribute to the current between the superconductors. This is also a part of the injected electric current and does have no effect on the Josephson current. In contrast, the current component $j_{2,2}$ does not change its sign at the superconductor interfaces, and therefore represents the nonvanishing current flow through the junction. This term reveals an electrically controllable ϕ_0 -junction that we discuss in detail in the next subsection.

C. Controllable ϕ_0 -Josephson junction

The Josephson current flowing through the junction is expressed by Eq. (52b). The ground state of the system corresponds to zero current and it is apparent that this condition is satisfied at a nonzero phase difference between the superconducting leads namely at: $\phi = \chi = -4d_I h/\alpha$. According to the previous section, this anomalous phase shift is proportional to the voltage bias V between the superconducting terminals and the additional normal injector electrode. Therefore, the ϕ_0 ground state of the Josephson junction can be experimentally switched on and off by simply controlling the quasiparticle flow injection.

The quasiparticle injection controls not only the anomalous phase shift but also modifies the critical current of the Josephson junction. To illustrate this fact, we plot the corresponding critical current in Fig. 3 vs the voltage difference V for two different lengths of the TI region. The panel (a) exhibits the critical supercur-

rent vs the voltage difference where the junction thickness is $d = \xi_s$ while in panel (b) we set $d = 3.0\xi_s$. Here $\xi_s = \sqrt{\hbar D/|\Delta|}$ and this quantity is a superconducting coherence if the diffusion constant in the superconducting terminals and TI surface are equal. The current is measured in units of $-\sigma_N \Delta \xi_s e^{-1} \gamma^{-2}$. As seen, the nonequilibrium conditions, generated by the quasiparticles injection into the TI region, strongly alter the critical Josephson current flowing through the junction and can result in supercurrent reversals. We see that the critical current in a junction of thickness $d = \xi_s$ shows a single sign reversal while larger thicknesses can result in multiple sign reversals. Note that sign reversals of supercurrent due to the quasiparticles injection was also theoretically predicted in conventional diffusive metals^{43,44} and observed in experiments⁴⁵⁻⁴⁷. However, the novelty of our finding is a transition between χ and $\chi + \pi$ ground states rather than $0 \rightarrow \pi$ transition. Our proposal offers new venues to explore more about the superconductor-topological insulator heterostructures, identify their topological char-

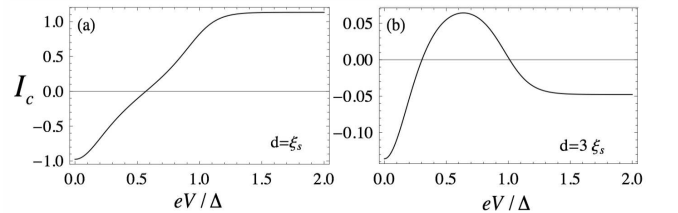


FIG. 3. Critical supercurrent as a function of voltage difference between the normal injector electrode and superconducting terminals eV/Δ . In panel (a) we set the length of TI region equal to $d = \xi_s$ while in panel (b) $d = 3\xi_s$.

acteristics, and potentially utilize them for technological functionalities.

V. CONCLUSIONS

Motivated by recent experiments on hybrid structures of topological insulator-superconductor^{17–22}, we have generalized the quasiclassical approach using the Keldysh technique to disordered TI-S heterostructures under nonequilibrium situations. We study the coupling between electric charge current and spin polarization on the surface of a three dimensional TI. Utilizing the generalized approach, we propose a setup to experimentally achieve a controllable ϕ_0 -Josephson junction solely by means of a quasiparticle flow injection without resorting to an external magnetic field or ferromagnetic element. The quasiparticles should be injected into the TI surface from the middle of junction by applying a voltage difference V between a normal injector electrode and the superconducting terminals. The quasiparticle currents in

opposite directions generate a Zeeman-like domain with opposite orientations. We show that the Zeeman-like term is directly coupled to the Josephson phase difference and shifts the junction ground state into arbitrary values by means of V and uncover the influences of V on critical supercurrent. We also discuss direct magnetoelectric effect that appears in response to a dc Josephson current on the surface channels and show that the induced electron spin polarization is much more pronounced than that of a conventional spin orbit coupled material. Our work demonstrates the great potential of topological insulators in magnetoelectrics by superconducting hybrid configurations due to the strong spin-momentum locking property of TI surface channels.

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